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Casimir stress for cylindrical shell in de-Sitter space

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Abstract

The Casimir stress on cylinderical shell in de Sitter background for massless scalar field satisfying Dirichlet boundary conditions on the cylinder is calculated. The metric is written in conformally flat form to make maximum use of the Minkowski space calculations. Different cosmological constants are assumed for the space inside and outside the cylinder to have general results applicable to the case of cylindrical domain wall formations in the early universe.

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1 Introduction

The Casimir effect is one of the most interesting manifestations of nontrivial properties of the vacuum state in quantum field theory [1,2]. Since its first prediction by Casimir in 1948 [3] this effect has been investigated for different fields having different boundary geometries [4-7]. The Casimir effect can be viewed as the polarization of vacuum by boundary conditions or geometry. Therefore, vacuum polarization induced by a gravitational field is also considered as Casimir effect. The types of boundary and conditions that have been most often studied are those associated to well known problems, e.g. plates, spheres, and vanishing conditions, perfectly counducting conditions, etc. The cylindrical problem with perfectly counducting conditions was first considered in [8], for a recent study ref.[9, 10].
In the context of hot big bang cosmology, the unified theories of the fundamental interactions predict that the universe passes through a sequence of phase transitions. These phase transitions can give rise to string structures determined by the topology of the manifold M of degenerate vacuua [11, 12, 13]. If M is disconnected, i.e. if $\pi(M)$ is nontrivial, then one can pass from one ordered phase to the other only by going through a domain wall. If M has two connected components, e.g. if there is only a discrete reflection symmetry with $\pi_0(M) = Z_2$, then there will be just two ordered phase separated by a domain wall.
The time evolution of topological defects have played an important role in many branches of physics, e.g., vortices in superconductors [14] and in superfluid [15], defects in liquid crystals [16], domain wall [17, 18], cosmic string [12, 13] and a flux tube in QCD [19].
Casimir effect in curved space-time has not been studied extensively. Casimir effect in the presence of a general relativistic domain wall is considered in [20] and a study of the relation between trace anomaly and the Casimir effect can be found in [21]. Casimir effect may have interesting implications for the early universe. It has been shown, e.g., in [22] that a closed Robertson-Walker space-time in which the only contribution to the stress tensor comes from Casimir energy of a scalar field is excluded. In inflationary models, where the dynamics of bubbles may play a major role, this dynamical Casimir effect has not yet been taken into account. Let us mention that in [23] we have investigated the Casimir effect of a massless scalar field with Dirichlet boundary condition in spherical shell having different vacuua inside and outside which represents a bubble in early universe with false/true vacuum inside/outside. In this reference the sphere have zero thickness. In another paper [24] we have extended the analysis to the spherical shell with nonvanishing thickness. Parallel plates immersed in different de Sitter spaces in- and out-side is calculated in [25].
Our aim is to calculate the Casimir stress on a cylindrical shell in which the coordinate z is absent with constant comoving radius having different vacuua inside and outside, i.e. with false/true vacuum inside/outside. Our model may be used to study the effect of the Casimir force on the dynamics of the cylindrical domain wall appearing in the simplest Goldston model. In this model potential of the scalar field has two equal minima corresponding to degenerate vacuua. Therefore, scalar field maps points at spatial infinity in physical space nontrivially into the vacuum manifold [26]. Domain wall structure occur at the boundary between these regions of space. One may assume that the outer regions of cylinder are in Λ_{out} vacuum corresponding to degenerate vacuua in domain wall configuration. In section two we calculate the stress on cylinder with Dirichlet bound-

ary conditions. The case of different de Sitter vacuua inside and outside the cylinder, is considered in section three. The last section concludes and summarizes the results.

2 Scalar Casimir effect for a cylindrical shell in flat space

In this section we calculate the Casimir energy of a massless scalar field in flat space which satisfies Dirichlet boundary condition on a cylinder.
The Casimir energy of a scalar field is given by

$$E_0 = \frac{1}{2}\zeta(s-1/2)\mu^{2s}, \tag{1}$$

where

$$\zeta(s) = \sum_n \lambda_n^{-s}, \tag{2}$$

is the zeta function of the corresponding Laplace operator. The arbitrary parameter μ has the dimension of a mass.
In our problem one can think of the circular problem in the plane, because in the problem which we consider here, the coordinate z is absent. The zeta function for interior region of circle is given by [27]

$$\zeta^{int}(s) = \frac{1}{a}[-\frac{16+\pi}{128\pi}(\frac{1}{s+1} + \ln a) + \frac{1}{8\pi}\frac{1}{s+1} + 0.00985], \tag{3}$$

and for exterior region we have

$$\zeta^{out}(s) = \frac{1}{a}[\frac{16-\pi}{128\pi}(\frac{1}{s+1} + \ln a) - \frac{1}{8\pi}\frac{1}{s+1} - 0.0085]. \tag{4}$$

The eigenvalues λ_n which enter the zeta function (2) are determined by

$$\nabla\varphi_n(x) = \lambda_n\varphi_n(x). \tag{5}$$

Therefore, using Eqs.(1,2), the Casimir energies inside and outside of cylinder are give by

$$E_{in} = \frac{1}{2\mu}\zeta^{int}(-1) = \frac{1}{2a\mu}[-\frac{16+\pi}{128\pi}(\frac{1}{\varepsilon} + \ln a) + \frac{1}{8\pi}\frac{1}{\varepsilon} + 0.00985], \tag{6}$$

$$E_{out} = \frac{1}{2\mu}\zeta^{out}(-1) = \frac{1}{2a\mu}[\frac{16-\pi}{128\pi}(\frac{1}{\varepsilon} + \ln a) - \frac{1}{8\pi}\frac{1}{\varepsilon} - 0.0085]. \tag{7}$$

Each of the energies for inside and outside of the cylinder is infinite, and cutoff dependent. The Casimir energy E is the sum of Casimir energies E_{in} and E_{out} for inside and outside of the cylinder.

$$E = E_{in} + E_{out} = \frac{1}{2a\mu}[\frac{-1}{64}(\frac{1}{\varepsilon} + \ln a) + 0.00135]. \tag{8}$$

As one can see, the Casimir energy E is also infinite and dependent to the cutoff ε. At this stage we introduce the classical energy for inside and outside separately and try to absorb divergent parts into these classical energies. This technique of absorbing an infinite quantity into a renormalized physical quantity is familiar in quantum field theory and quantum field theory in curved space [30]. Here we use a procedure similar to that of bag model [31, 32] (to see application of this renormalization procedure in Casimir effect problem in curved space refer to [23, 24]). The classical energy of cylinder in which the z coordinate is absent may be written as

$$E_{class} = \sigma a^2 + Fa + K + \frac{h}{a}. \tag{9}$$

The total energy of the cylinder inside and outside may be written as

$$\tilde{E}_{in} = E_{in} + E^{in}_{class} \tag{10}$$

$$\tilde{E}_{out} = E_{out} + E^{out}_{class}. \tag{11}$$

In order to obtain a well defined result for the Casimir energies inside and outside cylinder, we have to renormalize the parameter h of classical energy according to below

$$h^{in} \to h^{in} + \frac{1}{256\mu\varepsilon} \tag{12}$$

$$h^{out} \to h^{out} + \frac{1}{256\mu\varepsilon}. \tag{13}$$

Hence the effect of the vacuum fluctuation of scalar quantum field is to change, or renormalize parameter h of classical energy inside and outside of cylinder. Therefore, we rewrite Eqs.(10,11) as

$$\tilde{E}_{in} = E_{in} + \tilde{E}^{in}_{class} = \frac{1}{2a\mu}[0.00985 - \ln a(\frac{1}{128} + \frac{1}{8\pi})] + \frac{h^{in}}{a}, \tag{14}$$

$$\tilde{E}_{out} = E_{out} + \tilde{E}^{out}_{class} = \frac{1}{2a\mu}[-0.0085 + \ln a(\frac{-1}{128} + \frac{1}{8\pi})] + \frac{h^{out}}{a}. \tag{15}$$

We finally obtain for the total zero point energy of our system

$$\tilde{E} = \frac{1}{2a\mu}[0.00135 - \frac{\ln a}{64}]. \tag{16}$$

Once, the infinite terms have been removed from E in Eq.(8), the remainder is finite and will be called the renormalized Casimir energy. The Casimir stress on the cylinder due to the boundary conditions is then obtained

$$\frac{\bar{F}}{A} = \frac{-1}{2\pi a}\frac{\partial \tilde{E}}{\partial a} = \frac{1}{4\pi\mu a^3}[(0.00135 - \frac{\ln a}{64}) + \frac{1}{64}]. \tag{17}$$

3 Cylindrical shell in de Sitter space

Consider now a massless scalar filed in de Sitter space-time which satisfies Dirichlet boundary condition on a cylindrical shell. To make the maximum use of the flat space calculation we use the de Sitter metric in conformally flat form

$$ds^2 = \frac{\alpha^2}{\eta^2}[d\eta^2 - \sum_{i=1}^{3}(dx^i)^2], \tag{18}$$

where η is the conformal time

$$-\infty < \eta < 0. \tag{19}$$

The constant α is related to the cosmological constant as

$$\alpha^2 = \frac{3}{\Lambda}. \tag{20}$$

Now we consider the pure effect of vacuum polarization due to the gravitational field without any boundary conditions (to see such problem for spherical shell and parallel plate geometry refer to [23, 24, 25]). The renormalized stress tensor for massless scalar field in de Sitter space is given by [30, 33]

$$< T_\mu^\nu >= \frac{1}{960\pi^2\alpha^4}\delta_\mu^\nu. \tag{21}$$

The corresponding effective pressure is

$$P = - < T_1^1 >= - < T_r^r >= -\frac{1}{960\pi^2\alpha^4}, \tag{22}$$

valid for both inside and outside the cylinder. Hence the effective force on the cylinder due to the gravitational vacuum polarization is zero.
Now, assume there are different vacuua inside and outside corresponding to α_{in} and α_{out} for the metric (18). Now, the effective pressure created by gravitational part (22), is different for different part of space-time

$$P_{in} = - < T_r^r >_{in}= -\frac{1}{960\pi^2\alpha_{in}^4} = \frac{-\Lambda_{in}^2}{8640\pi^2}, \tag{23}$$

$$P_{out} = - < T_r^r >_{out}= -\frac{1}{960\pi^2\alpha_{out}^4} = \frac{-\Lambda_{out}^2}{8640\pi^2}. \tag{24}$$

Therefore the gravitational pressure over shell, P_g, is given by

$$P_g = P_{in} - P_{out} = \frac{-1}{8640\pi^2}(\Lambda_{in}^2 - \Lambda_{out}^2). \tag{25}$$

Now we consider the effective pressure due to the boundary condition. Under the conformal transformation in four dimensions the scalar field $\Phi(x, \eta)$ is given by

$$\bar{\Phi}(x, \eta) = \Omega^{-1}(x, \eta)\Phi(x, \eta), \tag{26}$$

with the conformal factor given by

$$\Omega(\eta) = \frac{\alpha}{\eta}. \tag{27}$$

And assuming a canonical quantization of the scalar field, and using the creation and annihilation operators $a_k^\dagger$ and a_k, the scalar field $\Phi(x, \eta)$ is then given by

$$\Phi(x, \eta) = \Omega(\eta)\sum_k [a_k \bar{u}_k(\eta, x) + a_k^\dagger \bar{u}_k^*(\eta, x)] \tag{28}$$

The vacuum states associated with the modes $\bar{u}_k$ defined by $a_k|\bar{0}\rangle = 0$, are called conformal vacuum. For the massless scalar field we are considering, the Green's function $\bar{G}$ associated to the conformal vacuum $|\bar{0}\rangle$ is given by the flat Feynman Green's function times a conformal factor [30, 34]. The two-point Green's function, $G(x, t; x', t')$, is defined as the vacuum expectation value of the time-ordered product of two fields

$$G(x, t; x', t') = -\imath\langle 0|T\Phi(x, t)\Phi(x', t')|0\rangle. \tag{29}$$

Given the above flat space Green's function, we obtain

$$\bar{G} = -\imath\langle\bar{0}|T\bar{\Phi}(x, \eta)\bar{\Phi}(x', \eta^{'})|\bar{0}\rangle = \Omega^{-1}(\eta)\Omega^{-1}(\eta^{'})G. \tag{30}$$

Using this Green's function we can obtain the Casimir stress inside and outside of the cylinder in de Sitter space

$$(\frac{\bar{F}}{A})_{in} = \frac{\eta^2}{\alpha_{in}^2}(\frac{F}{A})_{in}, \tag{31}$$

$$(\frac{\bar{F}}{A})_{out} = \frac{\eta^2}{\alpha_{out}^2}(\frac{F}{A})_{out}. \tag{32}$$

Now, using Eqs.(14,15,17) and considering the zero point energy inside and outside we can write

$$(\frac{\bar{F}}{A})_{in} = -\frac{1}{2\pi a}\frac{\eta^2}{\alpha_{in}^2}\frac{\partial\tilde{E}_{in}}{\partial a} = \frac{1}{4\pi\mu a^3}\frac{\eta^2}{\alpha_{in}^2}[0.00985 + (1 - \ln a)(\frac{1}{128} + \frac{1}{8\pi})], \tag{33}$$

$$(\frac{\bar{F}}{A})_{out} = -\frac{1}{2\pi a}\frac{\eta^2}{\alpha_{out}^2}\frac{\partial\tilde{E}_{out}}{\partial a} = \frac{1}{4\pi\mu a^3}\frac{\eta^2}{\alpha_{out}^2}[-0.0085 + (lna - 1)(\frac{-1}{128} + \frac{1}{8\pi})]. \tag{34}$$

Therefore, the vacuum pressure due to the boundary condition acting on the cylinder is given by

$$\begin{aligned} P_b &= (\frac{\bar{F}}{A})_{in} + (\frac{\bar{F}}{A})_{out} = \frac{\eta^2}{4\pi\mu a^3}[\frac{1}{\alpha_{in}^2}[0.00985 + (1 - \ln a)(\frac{1}{128} + \frac{1}{8\pi})] \\ &+ \frac{1}{\alpha_{out}^2}[-0.0085 + (1 - \ln a)(\frac{1}{128} - \frac{1}{8\pi})]]. \end{aligned} \tag{35}$$

The total pressure on the circle, P, is then given by

$$\begin{aligned} P &= P_g + P_b = \frac{1}{8640\pi^2}(\Lambda_{out}^2 - \Lambda_{in}^2) + \frac{\eta^2}{12\pi\mu a^3}((1 - \ln a)[\Lambda_{in}(\frac{1}{128} + \frac{1}{8\pi}) \\ &+ \Lambda_{out}(\frac{1}{128} - \frac{1}{8\pi})] + 0.00985\Lambda_{in} - 0.0085\Lambda_{out}). \end{aligned} \tag{36}$$

The η- or time-dependence of the pressure is intuitively clear due to the time dependence of the physical radius of cylinder. Total pressure, may be negative or positive, depending on the difference between the cosmological constant in the two parts of space-time. Given a false vacuum inside of the cylinder , and true vacuum outside, i.e. $\Lambda_{in} > \Lambda_{out}$, if $1 > \ln a$, then the gravitational part is negative, and tends to contract the cylinder. In contrast, the boundary part is positive and will lead to the repulsive force. Therefore, the total effective pressure on the cylinder may be negative, leading to a contraction of the cylinder. The contraction, however, ends for a minimum of radius of the cylinder, where both part of the total pressure are equal. For the case of true vacuum inside the cylinder and false vacuum outside, i.e $\Lambda_{in} < \Lambda_{out}$, the gravitational pressure is positive. In this case, boundary part can be negative or positive depending on the difference between Λ_{in} and Λ_{out}. Hence, the total pressure may be either negative or positive.

4 Conclusion

We have considered a cylinder in which the coordinate z is absent in de Sitter background with a massless scalar field, coupled conformally to it, satisfying the Dirichlet boundary conditions with constant comoving radius. Our calculation shows that for the cylindrical shell with false vacuum inside and true vacuum outside, the gravitational pressure part is negative, but the boundary pressure part is positive. In contrast for the case of true vacuum inside the cylinder and false vacuum outside, the gravitational pressure is positive, and boundary part can be negative or positive depending on the difference between cosmological constant inside and outside of cylinder. The result may be of interest in the case of formation of the cosmic cylinderical domain walls in early universe.

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